December 2017

Fondazione Eni Enrico Mattei

Working Paper

057.2017

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Mitigation, Innovation & Transformation Pathways
Series Editor: Massimo Tavoni

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Summary

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Keywords: Climate Change, Outgassing, Carbon Dioxide Removal, Integrated Assessment Model (DICE)

JEL Classification: Q53, Q54

The authors would like to thank the participants at the EAERE 2017 Conference, Athens, for very valuable suggestions. All errors are our own. The research leading to these results has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No 703399 for the project 'Robust Policy'.

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Optimal carbon dioxide removal in face of ocean carbon sink feedback*

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Abstract

Carbon dioxide removal (CDR) is a potentially important climate strategy for attaining low climate stabilization objectives. However, climate analysis has indicated a possible weakening of the ocean carbon sinks -the largest in the world- in relation to CDR deployment. Here, we provide an economic appraisal to assess the sensitivity of CDR and conventional abatement to CO_2 outgassing from the oceans. We develop a theoretical framework to study the impact of the ocean-to-atmosphere transfer on the optimal mitigation strategies under different regimes that control the relationship between CO_2 outgassing and the amount of CDR. We show that the optimal levels of emissions and CDR are correlated to the effectiveness of CDR expressed as a linear function of atmospheric concentrations. We incorporate this effect into an integrated assessment model of climate and economy (DICE model) and confirm the theoretical findings with numerical simulations. Further, we perform a sensitivity analysis to find the range of optimal abatement and CDR actions under different values of the CDR effectiveness coefficient.

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1 Introduction

The ultimate objective of the Paris agreement is to achieve stringent temperature targets as low as 1.5 and 2°C above the preindustrial level. However, these temperature targets require global cooperation and a fast and high decrease of the levels of the greenhouse gases (GHGs). Even with a significant reduction of the accumulation of the greenhouse gases, the integrated assessment models predict that the target of 1.5 and 2 °C would be impossible without additional measures besides traditional abatement (Edenhofer et al. (2014),Kriegler et al. (2016)).

Among alternative policy options that could help achieving safe temperature targets there is climate engineering -the deliberate intervention in the planetary environment of a nature and scale intended to counteract anthropogenic climate change and its impacts (Shepherd (2009)). There are two basic types of climate engineering : a) Carbon dioxide removal (CDR) techniques which remove CO_2 from the atmosphere and b) Solar Radiation Management (SRM) techniques that reflect a small percentage of the sun's light and heat back into space. The advantage of Carbon dioxide removal -also known as negative emissions technology- is that it addresses the root cause of climate change by removing greenhouse gases from the atmosphere. In this sense, it represents an enhanced form of mitigation.

In recent years the concept of negative emissions has been considered as an additional solution to abatement dealing with the problem of global warming. Model comparisons of integrated assessment models of energy, climate and the economy have shown that negative emissions are essential strategies in achieving 1.5 and 2 °C objectives (IPCC (2014); Kriegler et al. (2014); Rockström et al. (2017)). CDR allows to expand the admissible CO_2 budget, by relaxing initial mitigation efforts at the expenses of deeper emission cuts later in the century, often resulting in significant net negative emissions of several $GtCO_2/yr$.

Despite its importance in helping reconciling the historical and current emissions with the limited CO_2 budget required to achieve climate stabilization, CDR raises serious issues which might hinder its deployment below what technically feasible. A special issue about CDR in 2013 already indicated potentially critical elements of CDR, in terms of biological and geological constraints, impact on ecosystems, and governance (Tavoni and Socolow (2013)). Since then, other studies have further elaborated the potential risks associated with CDR (Fuss et al. (2014)).

A possible side effect from CDR is related to the disturbance of existing natural carbon sinks. Carbon sinks are removing as much as half of total anthropogenic greenhouse gas emissions emitted in the atmosphere, roughly equally split between oceans and land sinks. Just as CO_2 emissions into the atmosphere result in CO_2 passing from the atmosphere to the ocean to sustain equilibrium at the ocean's surface, so too do flows from ocean to atmosphere accompany atmospheric removal.

Climate scientists have warned that the deliberate removal of CO_2 from the atmosphere will be accompanied by ocean outgassing of CO_2 and thus in a reduction of the net emissions removed from the atmosphere (Cao and Caldeira (2010); Vichi et al. (2013); Jones et al. (2016)). Using an ocean-atmosphere carbon cycle model Vichi et al. (2013) investigate two different sets of experiments, one with the imposition of a reference atmospheric CO_2 concentration pathway and one with prescribed negative emissions and the atmospheric CO_2 let free to evolve according to the surface fluxes and the general circulation. They show that both actions are anticipated to release the anthropogenic carbon stored in the surface ocean, effectively increasing the required removal effort and that the additional negative emissions are expected to be lower when the CDR policy is driven by planned removal rates without prescribing a target atmospheric CO_2 concentration. Cao and Caldeira (2010) use an Earth system model to investigate the response of the coupled climate–carbon system to an instantaneous removal of all anthropogenic CO_2 from the atmosphere. In their simulations a one-time removal of all anthropogenic CO_2 from the atmosphere offsets less than 50 of the warming experienced at the time of removal and to maintain atmospheric CO_2 concentrations at pre-industrial levels for centuries, an additional amount of CO_2 equal to the original CO_2 captured would need to be removed over the subsequent 80 years. Moreover, they observe that to maintain atmospheric CO_2 and temperature at low levels, not only does anthropogenic CO_2 in the atmosphere need to be removed, but anthropogenic CO_2 stored in the ocean and land needs to be removed as well when it outgasses to the atmosphere. Finally, Jones et al. (2016) show that under low carbon pathways natural sinks will significantly weaken, hindering the effectiveness of negative emissions technologies and therefore increasing their required deployment to achieve a given climate stabilisation target.

Given the fundamental role and sheer size of ocean carbon sinks, the feedback between CO_2

removal from CDR and the response of natural carbon sinks should be accounted for when evaluating the potential of CDR as a climate strategy. Yet, existing studies by integrated assessment modelers and climate economists have mostly disregarded this issue, by assuming carbon sinks will not be affected by the amount of negative emissions and background concentrations. One exception is Chen and Tavoni (2013), where the authors using an integrated assessment model (IAM) find that the negative feedback of excess outgassing can significantly reduce the scope for CDR for a given climate temperature objective.

In this paper, we address the question of how the inclusion of CDR as an option against global warming, changes the optimal path of carbon emissions and how this option affects the ocean sinks and the outgassing process. We develop a linear quadratic model to analyze the optimal dynamic mix of abatement and CDR strategies in the presence of the outgassing effect.

In contrast to previous contributions (Vichi et al. (2013), Cao and Caldeira (2008), Cao and Caldeira (2010)), we provide an explicit analytical solution to the optimization problem that clarifies the structure of the optimal CDR and abatement policies. Moreover to the best of our knowledge, our paper is the first to completely characterize and interpret the dynamics of the optimal CDR levels and the outgassing effect and attach a statistically meaningful, as well as analytically tractable, coefficient on the ocean-to-atmosphere transfer, which basically represents the CDR effectiveness. These insights prove especially useful in the paper's numerical exercise.

We develop a theoretical model to represent the carbon cycle in two consecutive time periods and we solve the cost benefit optimization problem for three different regimes. In the first regime we assume that the only policy option is abatement and the global planner has no access to CDR. In the second regime we include CDR as a policy against climate change but we assume that there is a fixed level of outgassing from the oceans. Finally in the last regime we include the feedback on the ocean sink by linking the effectiveness of CDR to the level of atmospheric concentrations. We show that when there is the CDR option in the mitigation policy mix, but with a coefficient referring to the outgassing amount depending linearly on the carbon stock in the atmosphere, the optimal decision is to lower abatement as CDR acts substitutes for it. However, in this case the optimal policy is to do less abatement and instead use more CDR as the effectiveness of CDR increases.

We complement the theoretical analysis with numerical exercises. To this end, we modify an integrated assessment model to incorporate the linkage between outgassing and carbon dioxide removal, mimicking the theoretical model. The dynamic integrated climate-economy (DICE) model has been widely used to study climate change and optimal climate policy. A unique contribution of this paper in terms of methods is to modify the original DICE model by including a choice variable for the level of CDR in addition to DICE's choice variable for the intensity of abatement along with a two box carbon cycle dynamic. Thus, in addition to choosing an optimal abatement path, our model solves for an optimal CDR path. Our main finding is that by the end of this century, the optimal deployment of CDR not only removes all emissions but also reduces the existing carbon concentration.

In Section 2 we introduce and solve the theoretical model under three different regimes. In sections 3 and 4 we perform the numerical simulation using DICE. In section 3 we summarize the modification to DICE model and the sensitivity analysis is being introduced in section 4. Finally, in Section 5 we present our concluding remarks.

2 The Theoretical Framework

2.1 The model

We use a linear quadratic model to analyze in a cost benefit framework the optimal dynamic mix of abatement and CDR strategies in the presence of CO_2 outgassing. We investigate the optimal anthropogenic intervention into the carbon cycle in the light of global warming as a social planner's problem. Thus, the planner needs to determine the global optimal amount of emissions and CDR effort with related outgassing released to the atmosphere. The global decision maker's objective is to maximize the global welfare by choosing the optimal emissions (E_t) and CDR (R_t) levels in each time period. The relationship between emissions, economic output and consumption is modeled through a reduced form utility function depending on emissions. Within our linear-quadratic framework, the utility-emission function is given by the quadratic function

$$U(E_t) = E_t - \frac{1}{2}aE_t^2 \tag{1}$$

where a is a parameter indicating the slope of the private marginal benefits from emissions and can be regarded as reflecting the strength of diminishing returns.
As for the CDR implementation, we assume that generates additional costs in the social welfare function at any instant in time. We use a simple quadratic cost function for the cost of CDR in period t, which is strictly increasing and convex :

$$C(R_t) = \frac{1}{2}c_R R_t^2 \tag{2}$$

where c_R represents the marginal cost of CDR.
Damages from the social costs of global warming due to the cumulative emissions, represented by a convex, quadratic in our case, function of the atmospheric carbon stock above pre-industrial levels:

$$D(M_t) = \frac{1}{2}\delta_M M_t^2 \tag{3}$$

where δ_M represents the marginal damages from global warming and M_t is the carbon stock in the atmosphere at time t.

The carbon cycle is

$$M_{t+1} = M_t + E_t - \beta_0 S_t - (\gamma_0 + \gamma_1 M_t)R_t \tag{4}$$

where S_t is the carbon stock in the oceans at time t. Equation (4) constitutes the model representation of the atmosphere and describes the dynamics of the global carbon cycle as a consequence of the anthropogenic intervention. We assume that there is a net transfer of carbon between the atmosphere and the ocean. The downward flux of carbon from the atmosphere to the ocean (ocean uptake) is represented by the factor $\beta_0 S_t$ and the upwards flux of carbon from the ocean to the atmosphere (outgassing) is represented by the factor $(1 - (\gamma_0 + \gamma_1 M_t))R_t$. These two fluxes describe the net transfer between the atmosphere and the ocean and there is a net flux between these two layers represented by the difference between the relative stock sizes. The carbon stock in the atmosphere is much smaller than the carbon stock in the ocean, thus β_0 is the proportionality factor to scale the stock of carbon in the ocean with respect to the atmosphere. The effectiveness of CDR in removing CO_2 from the atmosphere when accounting for ocean-atmosphere transfers is given by $(\gamma_0 + \gamma_1 M_t)$ in equation (4). Thus, the actual emissions reductions brought about by negative emissions are equal the effectiveness term by the level of CDR (R_t). The coefficient of the fixed CDR effectiveness is γ_0 and adjusts the amount of carbon being absorbed by the oceans[1]. While the coefficient referring to the CDR effectiveness depending linearly on the carbon stock in the atmosphere (M_t) and the level of CDR (R_t) is γ_1[2]. Thus, an additional level of the CDR would be required to compensate for ocean outgassing[3], in addition to the planned level required if outgassing did not exist. This formulation is in line with the results of climate science which predicts more outgassing (e.g. lower effectiveness of the sink) in the presence of lower background carbon concentrations, and a rough proportionality to the CDR deployed (Jones et al. (2016)).
The whole model can be thus formulated as

$$\underset{E_t, R_t}{Max}\; W(E_t, R_t) = U(E_t) - C(R_t) - D(M_{t+1}) \tag{5}$$

$$s.t.\ (4) \tag{6}$$

We introduce a two-period ($t = 1, 2$) model of abatement and CDR. Therefore the optimization model can be expressed as maximization of the utility function in the first period:

$$W(E_1, R_1) = U(E_1) - C(R_1) - D(M_2) \tag{7}$$

[1]By the term coefficient of the fixed CDR effectiveness, we mean that this coefficient is not related to the carbon stock in the atmosphere (M_t), as it is obvious from equation (4).

[2]From now on we will refer to γ_0 as the "fixed CDR effectiveness coefficient" and to γ_1 as the "linear CDR effectiveness coefficient".

[3]The connection between CDR effectiveness and outgassing is straightforward. If we assume that CDR is highly effective, at the same time we admit that the outgassing effect is low.

The welfare depends on the utility of the emissions, minus the cost of CDR in the first period and the damages from the carbon atmospheric stock in the next period. The Carbon Stock in the atmosphere (M_2) in the second period is expressed by:

$$M_2 = M_1 + E_1 - \beta_0 S_1 - (\gamma_0 + \gamma_1 M_1) R_1 \tag{8}$$

In the second period the global planner observes the carbon stocks in the atmosphere without taking any new action with respect to abatement or CDR effort. Thus, if we substitute (1), (2) and (3) into (7), we will have the following maximization problem

$$\max_{E_1, R_1} W(E_1, R_1) = E_1 - \frac{1}{2} a E_1^2 - \frac{1}{2} c_R R_1^2 - \frac{1}{2} \delta_M M_2^2 \tag{9}$$

2.2 Different Regimes

We solve the optimization problem for three different regimes. In the first regime we assume that the only policy option is abatement and the global planner has no access to the CDR technology. In the second regime we include the CDR option as a policy against climate change but we assume that there is constant outgassing from the oceans and in the last regime we assume that the CDR effectiveness coefficient is linearly linked to the CDR effort.

2.2.1 First Regime: Only Abatement

In this regime the only available option for the policy maker against climate change is abatement and the objective is to maximize welfare with respect to the emissions level.
The maximization problem is

$$\max_{E_1} W(E_t) = E_1 - \frac{1}{2} a E_1^2 - \frac{1}{2} \delta_M M_2^2 \tag{10}$$

subject to

$$M_2 = M_1 + E_1 - \beta_0 S_1 \tag{11}$$

From the FOCs:

$$\frac{\partial W_1}{\partial E_1} = 0 \Rightarrow E_t^* = \frac{1 - \delta_M y_1}{a + \delta_M}, \tag{12}$$

where $y_1 = M_1 - \beta_0 S_1 > 0$. Optimal emissions level depends negatively on the marginal damages from global warming and it is clear that the higher the damages are the lower the optimal emission level will be.
Differentiating the optimal emissions (E^*) and the atmospheric stock of carbon (M_2) with respect to the parameter of sensitivity of the ocean sink (β_0) respectively, yields

$$\frac{\partial E_t^*}{\partial \beta_0} = \frac{\delta_M S_1}{a + \delta_M} \Rightarrow \frac{\partial E_t^*}{\partial \beta_0} > 0 \tag{13}$$

$$\frac{\partial M_2}{\partial \beta_0} = -\frac{a S_1}{a + \delta_M} \Rightarrow \frac{\partial M_2}{\partial \beta_0} < 0 \tag{14}$$

From (13), (14) it is clear that the optimal level of emissions (E_t^*) is increasing in the parameter of the sensitivity of the ocean sink (β_0). Thus, if the sink is stronger and absorbs a larger amount of the emissions, this leads to a higher optimal level of emissions. For the atmospheric stock of carbon we obtain that a stronger oceanic sink (β_0) will lead to a lower carbon stock in the atmosphere (M_2).

2.2.2 Second Regime: Abatement & CDR (constant effectiveness and Outgassing)

Suppose that the policy maker has the CDR technology along with abatement and wants to maximize the global welfare with respect to these two options. There is a fixed outgassing from the oceans in this case. The welfare is

$$W(E_t, R_t) = U(E_t) - C(R_t) - D(M_{t+1}) \tag{15}$$

and the carbon cycle:

$$M_2 = M_1 + E_1 - \beta_0 S_1 - \gamma_0 R_1 \tag{16}$$

Thus the optimization problem can be formulated as

$$\underset{E_1, R_1}{max} W(E_1, R_1) = E_1 - \frac{1}{2} a E_1^2 - \frac{1}{2} c_R R_1^2 - \frac{1}{2} \delta_M M_2^2 \tag{17}$$

From the FONCs:

$$\frac{\partial W}{\partial E} = 0 \Rightarrow E_t^* = \frac{\delta_M \gamma_0^2 + c_R(1 - \delta_M y_1)}{a\delta_M \gamma_0^2 + c_R(a + \delta_M)} \tag{18}$$

$$\frac{\partial W}{\partial R} = 0 \Rightarrow R_t^* = \frac{\delta_M (1 + a y_1)\gamma_0}{a\delta_M \gamma_0^2 + c_R(a + \delta_M)} \tag{19}$$

where $y_1 = M_1 - \beta_0 S_1 > 0$. The optimal emission and CDR levels depends on the sum of the marginal damages from global warming (δ_M) and ocean acidification (c_R).
Differentiating optimal emissions and CDR levels with respect to the fixed CDR effectiveness coefficient (γ_0)

$$\frac{\partial E_t^*}{\partial \gamma_0} = \frac{2{\delta_M}^2 \gamma_0 c_R (1 + a y_1)}{(a\delta_M \gamma_0^2 + c_R(a + \delta_M))^2} \Rightarrow \frac{\partial E_t^*}{\partial \gamma_0} > 0 \tag{20}$$

$$\frac{\partial R_t^*}{\partial \gamma_0} = \frac{\delta_M (1 + a y_1)(a c_R + \delta_M (c_R - a{\gamma_0}^2)}{(a\delta_M \gamma_0^2 + c_R(a + \delta_M))^2} \Rightarrow \frac{\partial R_t^*}{\partial \gamma_0} > 0 \tag{21}$$

Both optimal emission level (E_t^*) and optimal CDR level (R_t^*) [4] are increasing in the effectiveness of CDR (γ_0) (i.e decreasing in the outgassing). Thus, the optimal decision is such that more (less) emissions and CDR happen the more (less) effective will the CDR be, when the outgassing effect is not directly related to the carbon stock in the atmosphere. However, this result is subjective to the strong assumption that the ocean sink is not affected by the CDR effort. Hence, in this regime we assumed that the natural process of the outgassing is not disrupted by the change in the stock of carbon in the atmosphere due to CDR.

To study the impact of the fixed CDR effectiveness coefficient (γ_0) on the optimal level of carbon concentration, we compute

$$\frac{\partial M_t^*}{\partial \gamma_0} = -\frac{2a\gamma_0 c_R \delta_M (1 + a y_1)}{(a\delta_M \gamma_0^2 + c_R(a + \delta_M))^2} \Rightarrow \frac{\partial M_t^*}{\partial \gamma_0} < 0 \tag{22}$$

The negative sign of (22) means that the rate of change in optimal level of effective CDR ($\gamma_0 \times R^*$) is faster than the rate of change in optimal emissions, thus more CDR is needed to reduce the carbon concentration in the atmosphere.

On the other hand, it is obvious that the higher the cost of CDR is, the lower the level of CDR that countries are willing to uptake will be.[5]

$$\frac{\partial R_t^*}{\partial c_R} = -\frac{\delta_M (a + \delta_M)(1 + a y_1)}{(a\delta_M {\gamma_0}^2 + a(\delta_M + c_R))^2} \Rightarrow \frac{\partial R_t^*}{\partial c_R} < 0 \tag{23}$$

2.2.3 Third Regime: Abatement & CDR (Outgassing linear in the CDR)

In this case we assume that the outgassing is a linear function of the CDR effort. This way we directly link the use of CDR methods to the perturbation of the natural oceanic sink.

$$W(E_t, R_t) = U(E_t) - C(R_t) - D(M_{t+1}) \tag{24}$$

The carbon cycle:

[4] We assume that the cost of CDR is higher than the cost of abatement ($c_R > a$) and for $0 < \gamma_0 < 1$, the term $(c_R - a{\gamma_0}^2)$ in 21 is positive.

[5] For $c_R \to \infty$ the optimal solution for the emissions is the same as in the first regime without the CDR policy.

$$M_2 = M_1 + E_1 - \beta_0 S_1 - (\gamma_0 + \gamma_1 M_1) R_1 \tag{25}$$

The maximization problem

$$\underset{E_1, R_1}{max} W(E_1, R_1) = E_1 - \frac{1}{2} a E_1^2 - \frac{1}{2} c_R R_1^2 - \frac{1}{2} \delta_M M_2^2 \tag{26}$$

From the FONCs:

$$\frac{\partial W}{\partial E} = 0 \Rightarrow E_t^* = \frac{\delta_M (\gamma_0 + M_1 \gamma_1)^2 + c_R (1 - \delta_M y_1)}{a \delta_M (\gamma_0 + M_1 \gamma_1)^2 + c_R (a + \delta_M)} \tag{27}$$

$$\frac{\partial W}{\partial R} = 0 \Rightarrow R_t^* = \frac{\delta_M (1 + a y_1)(\gamma_0 + M_1 \gamma_1)}{a \delta_M (\gamma_0 + M_1 \gamma_1)^2 + c_R (a + \delta_M)} \tag{28}$$

where $y_1 = M_1 - \beta_0 S_1 > 0$.

In this case the analysis will be focused on the outgassing effect and how it affects the optimal policy decisions. By differentiating optimal emissions and CDR levels with respect to (γ_0), we have:

$$\frac{\partial E_t^*}{\partial \gamma_0} = \frac{2 \delta_M^2 c_R (1 + a y_1)(\gamma_0 + M_1 \gamma_1)}{(a \delta_M (\gamma_0 + M_1 \gamma_1)^2 + c_R (a + \delta_M))^2} \Rightarrow \frac{\partial E_t^*}{\partial \gamma_0} > 0 \tag{29}$$

$$\frac{\partial R_t^*}{\partial \gamma_0} = \frac{\delta_M (1 + a y_1)(\delta_M c_R + a(c_R - \delta_M (\gamma_0 + M_1 \gamma_1)^2)}{\left(a \delta_M (\gamma_0 + M_1 \gamma_1)^2 + c_R (a + \delta_M)\right)^2} \Rightarrow \frac{\partial R_t^*}{\partial \gamma_0} > 0 \tag{30}$$

From Equations (29) and (30) it follows that the dynamics of the optimal emissions and CDR are not disturbed by the introduction of the linear relation between CDR and outgassing. Thus, the more effective CDR is, the more the countries will emit and use it.

By differentiating optimal emissions and CDR levels with respect to the linear CDR effectiveness coefficient (γ_1) we have

$$\frac{\partial E_t^*}{\partial \gamma_1} = \frac{2 \delta_M^2 c_R M_1 (1 + a y_1)(\gamma_0 + M_1 \gamma_1)}{\left(a \delta_M (\gamma_0 + M_1 \gamma_1)^2 + c_R (a + \delta_M)\right)^2} > 0 \tag{31}$$

$$\frac{\partial R_t^*}{\partial \gamma_1} = \frac{(\delta_M M_1 (1 + a y_1))\left(a c_R + \delta_M (c_R - a(\gamma_0 + M_1 \gamma_1)^2)\right)}{\left(a \delta_M (\gamma_0 + M_1 \gamma_1)^2 + c_R (a + \delta_M)\right)^2} > 0 \tag{32}$$

The dynamics for the optimal emissions and CDR levels are at the same direction, which means that as the correlation between the CDR effectiveness and the outgassing effect is strong, the optimal policy to emit more and to use more CDR when CDR is more effective (e.g. when outgassing is smaller). This strategy will be weakened in the case of a high implementation cost of CDR.

$$\frac{\partial R_t^*}{\partial c_R} = -\frac{\delta_M (a + \delta_M)(1 + a y_1)(\gamma_0 + M_1 \gamma_1)}{\left(a \delta_M (\gamma_0 + M_1 \gamma_1)^2 + c_R (a + \delta_M)\right)^2} < 0 \tag{33}$$

3 Numerical Simulation Model

In this section, we extend our analysis by modifying an integrated assessment model to incorporate the linkage between outgassing and carbon dioxide removal. The dynamic integrated climate-economy (DICE) model has been widely used to study climate change and optimal climate policy.

First we provide a brief qualitative description of the DICE model, and in the following sections we describe our modifications to the standard DICE model to incorporate CDR and outgassing. Appendix A of this paper provides more details on the model, including all of the model's equations and parametrization, and an extensive description of our solution algorithm.[6]

As in the 2013 version of DICE, our model is a finite horizon dynamic model with 60 time periods (300 years). It includes a representative agent model of the economy with exogenous technological growth. In each period (5 years), an existing capital stock is used as an input to an aggregate production function of labor, capital, and technological change. For simplification we

[6]See also Nordhaus (2008) for a summary of the model's assumptions and equations.

assume an exogenous, fixed savings rate: the representative consumer saves a fixed fraction of net output and consumes the rest.[7]

Carbon emissions are linked to economic production and accumulate in the atmosphere over time. A portion of atmospheric carbon stock sinks into the ocean. The radiative forcing - the difference between incoming short-wave radiation and outgoing long-wave energy (heat) - is a function of atmospheric carbon stock. Global temperature is a function of radiative forcing and past temperatures.

The economic and climate models are "integrated" together in that increasing global temperatures reduce net economic output. The gap between gross and net output is an increasing function of temperature, called the damage function. These damages can be avoided by spending on abatement to reduce emissions, and the cost of abatement is calibrated based on engineering and econometric studies.

The model can be used to calculate optimal climate abatement policy, which maximizes total discounted net consumption by comparing the costs of abatement with the damages from temperature growth. Optimal policy can be expressed by the optimal amount of abatement in each period as a percentage of emissions abated, a_t, or by the optimal carbon price in each period.

3.1 Summary of Modifications to DICE

Here we briefly summarize our modifications to DICE. These are based on the modifications in Heutel et al. (2015), and more detail is available there, as well as in this paper's appendix. There are four modifications made to DICE to incorporate CDR and outgassing.

3.1.1 Carbon cycle

In the original DICE model, the climate system consists of a dynamic model of carbon concentration based on a three box system (i.e. atmosphere, upper ocean, and lower ocean layers). We replace this model with a two box model of atmosphere and ocean.

3.1.2 CDR action

We include a choice variable for the level of CDR, r_t in addition to DICE's choice variable for the intensity of abatement, a_t. Thus, in addition to choosing an optimal abatement path, our model solves for an optimal CDR path. While a_t is the proportion of emissions that are abated and is between 0 and 1, r_t is the level of atmospheric concentration that is removed.

3.1.3 CDR's Effect on outgassing

CDR affects the outgassing from the oceans to Earth's atmosphere by reducing atmospheric concentration directly and therefore reducing the temperature. The process of removing CO_2 from the atmosphere is however, not perfect and the net carbon removal can be found by taking into account the effectiveness of CDR. The CDR effectiveness depends on the cumulative emissions in the atmosphere as expressed in Equation 4. We calibrate the parameter values of CDR effectiveness (γ_0 and γ_1), using the setting of experiments reported in Jones et al. (2016). Applying these numbers in our analytical model with linear outgassing, gives us the estimates for parameters $\gamma_0 = 0.5000$ and $\gamma_1 = 0.0002$. We use these values in calibrating our model. We then perform a sensitivity analysis over the values of γ_1.

3.1.4 CDR Implementation Cost

Our specification of implementation costs is analogous to DICE's specification of the cost of abatement. It is a convex (quadratic) function of the amount of emissions removed by the level of CDR, r_t. Abatement costs are expressed as a fraction of gross output in the DICE model. Here, we assume that CDR costs are independent of the size of the economy and can be expressed as independent costs. We further assume that the cost of abatement and CDR decreases over time at the same rate. It means that the CDR cost curve has the same slope as the abatement cost curve. It follows a cost reduction pattern over time that is proportional to the reduction in the abatement costs. At the start point of the cost curve we assume that the cost of removing all emissions through CDR is five times greater than the cost of abating 100% of emissions.

[7] In practice, when savings is allowed to be endogenous the savings rate only varies slightly from this fixed value.

4 Simulation Results and Discussion

In this section we conduct a sensitivity analysis that allow us to investigate the role of outgassing parameter in optimal decision making about abatement and CDR levels. we discuss the simulation model sensitivity analysis results for four values of outgassing parameter and we analyze the optimal climate policy portfolio of abatement and CDR. We calculate the resulting outgassing from the oceans and the GDP loss from ignoring CDR.

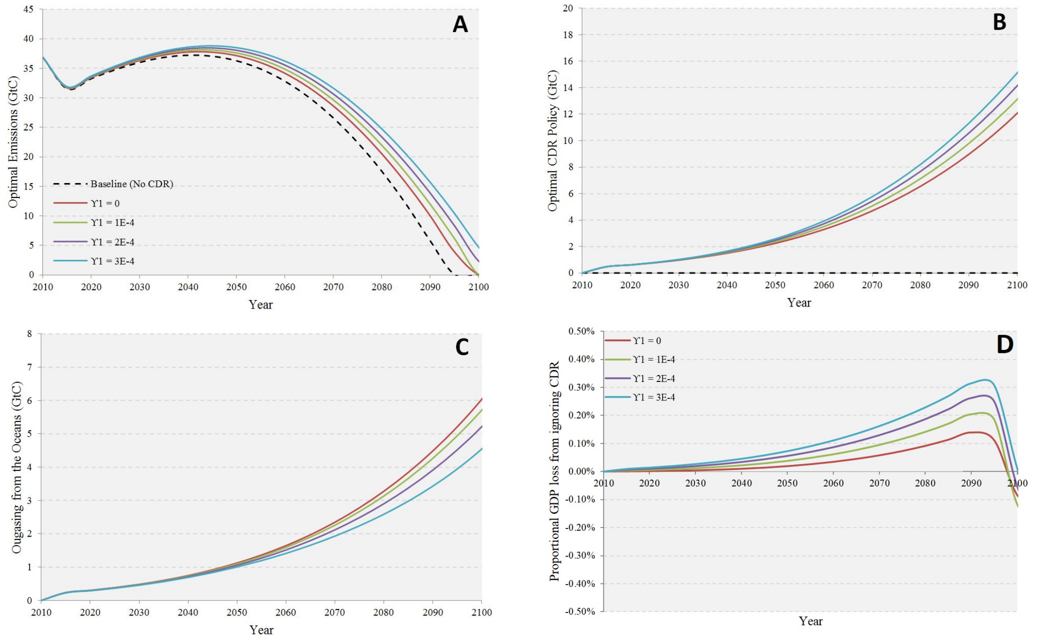

Figure 1: Climate outcomes under different values of the outgassing parameter.

Figure 1 shows the main results of our analysis using the modified DICE model with CDR action. Panel **A** shows the optimal emissions under each scenario. The case with no CDR is representing the results similar to the original DICE model but with a two box carbon cycle dynamic. We use this case as our baseline model. The next scenario ($\gamma_1 = 0$) is where CDR is implemented in the model and has maximum direct impact (minimum effectiveness). With $\gamma_0 = 0.5000$ this means that CDR is effective only for fifty percent, with half of CO_2 sequestered going back to the atmosphere due to outgassing. This case, represented by a red line, results in the most abatement and therefore the least level of emissions. The reason is that facing large outgassing from deploying CDR, the decision maker is more reluctant to use CDR as a substitute to abatement. This can be verified in panel **B**, where the lowest level CDR is deployed under the case with maximum direct outgassing from CDR ($\gamma_1 = 0$). The other three scenarios ($\gamma_1 = 0.0001$, $\gamma_1 = 0.0002$, and $\gamma_1 = 0.0003$) represent the cases where the outgassing from the ocean is decreasing as parameter γ_1 increases and the efficiency of CDR increases. This impact becomes more significant in determining the optimal policy. For the higher values of $gamma_1$, the optimal level of CDR increases (panel **B**), the optimal level of abatement decreases, and therefore the optimal level of emissions increases(panel **A**). This is due to the substitutability of abatement and CDR as both actions work in the same direction by reducing the carbon inventory and eventually controlling the atmospheric temperature.

In all scenarios, optimal emissions start increasing gradually to peak in around year 2050 and then decrease to near zero. The baseline case with no CDR, has the most stringent abatement policy. The emissions in this case peak below 38 GtC level while in the other cases with CDR, maximum emissions surpass this level. As abatement cost decreases over time, more abatement makes emission levels to fall fast and eventually reach the zero level in the baseline and very low values in all other cases (panel **A**). By design, CDR cost is decreasing over time proportional to the abatement cost, and therefore, CDR becomes cheaper and more valuable option in the future. After 100 years, the optimal level of CDR reaches to about 12 GtC in the case with maximum outgassing and 15 GtC in the case with the least outgassing effect. These levels are well above the

emission levels at the end of this century and therefore, indicate that CDR not only removes all emissions but also reduces the existing carbon concentration stock toward the end of the century. Panel **C** shows the outgassing from the ocean as a function of CDR deployment under each scenarios. By definition, there will be no outgassing in the baseline case without CDR. The most outgassing happens under the case with $\gamma_1 = 0$. In the three other cases with positive values of the parameter γ_1, there is a downward trend for outgassing as CDR is being used more effectively. Outgassing from the ocean reaches to about 6 GtC in 100 years in the case where there is the most outgassing effect ($\gamma_1 = 0$).
We compare the economic output from using CDR in different scenarios with the baseline case in panel **D**. This panel shows that ignoring CDR can result in up to about 0.3% GDP loss in the case with most effective CDR and minimum outgassing. The benefits of using CDR decreases as the outgassing effect increases. In all scenarios, the GDP loss from ignoring CDR peaks before year 2100 where abatement reaches its maximum rate and all emissions are removed (see panel **A**). Abatement cost in the DICE model is a power function of abatement rate. Once abatement rate reaches 100% it stays at that level, and therefore, abatement cost will increase with a smaller rate after that point. Since in all scenarios with CDR, abatement rate has not reached its maximum by year 2100, abatement costs will continue to grow faster after the peak point, compared to the baseline case. This explains the decline in GDP loss after the peak in panel **D**.

5 Conclusions

In this paper we have assessed the interplay between CDR and abatement along with the question on how the option of CDR affects the ocean sinks and the outgassing process. We have derived the optimal CDR and abatement levels using an analytical economic model and have verified our findings by using a modified integrated assessment model (DICE).

If CDR is not an option the optimal decision for the global planner is to emit less as abatement is the only option to control and reduce climate damages. When CDR is allowed in the policy mix, but with constant outgassing effect, the optimal level of emissions increases as CDR and abatement are substitutable. Finally, when the outgassing depends on how much CDR the decision maker uses, the optimal policy depends on how effectively CDR can be deployed. Outgassing cancels some of the benefits from CDR by releasing more carbon dioxide in to the atmosphere. This mechanism forces the optimal CDR to a lower level when it is less effective. Our main finding from the numerical exercise is that ignoring CDR can result in up to about 0.3% GDP loss in the case with highly effective CDR deployment. Thus, if we take into account the outgassing effect, the benefits of using CDR decreases as the outgassing effect increases. Our results also indicate that by the end of this century, CDR can potentially offset not only all emissions but also reduce the stock of existing carbon concentration.

Our study highlights the important link between the ocean flux and the optimal level of CDR. As climate engineering is gaining more popularity among scientific and climate change policy communities, it is necessary to evaluate the relationship between all components of the climate-economy system. We hope that our study leads the way in a more comprehensive approach to climate change policy options and their interactions with climate system.

References

Bony, S., G. Bellon, D. Klocke, S. Sherwood, S. Fermepin, and S. Denvil
2013. Robust direct effect of carbon dioxide on tropical circulation and regional precipitation. *Nature Geosci*, 6(6):447–451.

Brander, L. M., K. Rehdanz, R. S. J. Tol, and P. J. H. Van Beukering
2012. The economic impact of ocean acidification on coral reefs. *Clim. Change Econ.*, 03(01):1250002.

Cao, L. and K. Caldeira
2008. Atmospheric co2 stabilization and ocean acidification. *Geophysical Research Letters*, 35(19).

Cao, L. and K. Caldeira
2010. Atmospheric carbon dioxide removal: long-term consequences and commitment. *Environ. Res. Lett.*, 5(2):024011.

Chen, C. and M. Tavoni
2013. Direct air capture of co2 and climate stabilization: A model based assessment. *Climatic Change*, 118(1):59.

Edenhofer, O., R. Pichs-Madruga, Y. Sokona, E. Farahani, S. Kadner, K. Seyboth, A. Adler, I. Baum, S. Brunner, P. Eickemeier, et al.
2014. Climate change 2014: mitigation of climate change. *Contribution of working group III to the fifth assessment report of the intergovernmental panel on climate change*, 5.

Fuss, S., J. G. Canadell, G. P. Peters, M. Tavoni, R. M. Andrew, P. Ciais, R. B. Jackson, C. D. Jones, F. Kraxner, N. Nakicenovic, et al.
2014. Betting on negative emissions. *Nature Climate Change*, 4(10):850–853.

Heutel, G., J. M. Cruz, and S. Shayegh
2015. Solar Geoengineering, Uncertainty, and the Price of Carbon. Working Paper 21355, National Bureau of Economic Research. DOI: 10.3386/w21355.

IPCC
2014. *Climate Change 2014–Impacts, Adaptation and Vulnerability: Regional Aspects*. Cambridge University Press.

Jones, C. D., P. Ciais, S. J. Davis, P. Friedlingstein, T. Gasser, G. P. Peters, J. Rogelj, D. P. v. Vuuren, J. G. Canadell, A. Cowie, R. B. Jackson, M. Jonas, E. Kriegler, E. Littleton, J. A. Lowe, J. Milne, G. Shrestha, P. Smith, A Torvanger, and A. Wiltshire
2016. Simulating the Earth system response to negative emissions. *Environ. Res. Lett.*, 11(9):095012.

Kriegler, E., I. Mouratiadou, G. Luderer, N. Bauer, R. J. Brecha, K. Calvin, E. De Cian, J. Edmonds, K. Jiang, M. Tavoni, et al.
2016. Will economic growth and fossil fuel scarcity help or hinder climate stabilization? *Climatic Change*, 136(1):7–22.

Kriegler, E., J. P. Weyant, G. J. Blanford, V. Krey, L. Clarke, J. Edmonds, A. Fawcett, G. Luderer, K. Riahi, R. Richels, et al.
2014. The role of technology for achieving climate policy objectives: overview of the emf 27 study on global technology and climate policy strategies. *Climatic Change*, 123(3-4):353.

Nordhaus, W.
2008. A question of balance: Weighing the options on global warming policies (yale.

Nordhaus, W. and P. Sztorc
2013. Dice 2013r: Introduction and user's manual. *retrieved November*.

Nordhaus, W. D.
1993. Optimal Greenhouse-Gas Reductions and Tax Policy in the "DICE" Model. *The American Economic Review*, 83(2):313–317.

Pongratz, J., D. B. Lobell, L. Cao, and K. Caldeira
2012. Crop yields in a geoengineered climate. *Nature Clim. Change*, 2(2):101–105.

Rockström, J., O. Gaffney, J. Rogelj, M. Meinshausen, N. Nakicenovic, and H. J. Schellnhuber
2017. A roadmap for rapid decarbonization. *Science*, 355(6331):1269–1271.

Shepherd, J. G.
2009. Geoengineering the climate: science, governance and uncertainty.

Tavoni, M. and R. Socolow
2013. Modeling meets science and technology: an introduction to a special issue on negative emissions. *Climatic Change*, 118(1):1–14.

Vichi, M., A. Navarra, and P. G. Fogli
2013. Adjustment of the natural ocean carbon cycle to negative emission rates. *Climatic Change*, 118(1):105–118.

APPENDIX

A Details on the DICE model with CDR

In the paper, we briefly summarize the DICE model, our modifications to it, and our solution algorithm. Here, we provide more details. We modify the DICE model, first introduced by Nordhaus (1993). The model parameters and equations are from Nordhaus and Sztorc (2013). We have modified the DICE 2013R version of the model in order to include CDR action. This is a finite horizon model with 60 time steps. Each time step is 5 years, and the starting year is 2010. We model DICE similar to a Markov decision process but without uncertainty which includes a state space, an action space, a transition function, and a reward function.

- **State Space**
  The first modification to the DICE model is to replacing its 3-box carbon cycle with a 2-box carbon cycle. In order to do so, we have removed the upper ocean layer from the climate system in the original model and updated the carbon transfer coefficients between the remaining two layers in order to replicate the original results without CDR. The global climate-economy system can be defined as a state with five continuous variables: T_t^{at} is atmospheric temperature (degrees Celsius above preindustrial), T_t^{lo} is lower ocean temperature (degrees Celsius above preindustrial), M_t^{at} is the atmospheric concentration of carbon (Giga Tons of Carbon, GTC), M_t^{lo} is the concentration in deep oceans (GTC), and K_t is capital ($trillions). We define the state space as $S_t = \{T_t^{at}, T_t^{lo}, M_t^{at}, M_t^{lo}, K_t\}$.

- **Action Space**
  At each time step, a abatement action (control rate) a_t and a CDR action r_t are taken, which indicate the percentage reduction of GHG emissions and the removal level of atmospheric concentration, respectively. Both actions impose immediate costs but prevent the future damages of higher temperature. Taking actions a_t and r_t at any given state will determine the next state deterministically. Therefore the action space is defined as $a_t \in [0, 1]$ and $r_t \in [0, \bar{r}]$ where $\bar{r}$ is the theoretical maximum amount of concentration in the atmosphere that can be removed by CDR. As in the original DICE model, we assume that savings is fixed as a fraction of gross output and thus is not a choice variable.

- **Transition Functions**
  The gross economic output, Y_t, is calculated from the given level of technology, capital, and labor in the current state:
  $$Y_t = A_t \times K_t^{\epsilon} \times L_t^{1-\epsilon} \tag{A.1}$$
  where A_t is total factor of productivity and L_t is labor at time t. ϵ is the output elasticity of capital. The net output, Q_t, is calculated after subtracting climate change damages, and abatement and CDR costs from gross output:

  $$Q_t = \big(1 - D_t - C_a(a_t) - C_r(r_t)\big) \times Y_t \tag{A.2}$$
  $$D_t = \xi_1 \times T_t^{at} + \xi_2 \times (T_t^{at})^2 \tag{A.3}$$
  $$C_a(a_t) = \theta_1 \times a_t^{\theta_2} \tag{A.4}$$
  $$C_r(r_t) = \theta_1^r \times r_t^{\theta_2^r} \tag{A.5}$$

  where D_t includes damages from climate change that depend on the atmospheric temperature. The parameters ξ_1 and ξ_2 are the damage cost coefficients. The parameters θ_1 and θ_2 are the coefficients of the abatement cost function $C_a(a_t)$ and θ_1^r and θ_2^r are the coefficients of CDR cost function $C_r(r_t)$. In the DICE model the coefficient θ_1 is a decreasing function in time which indicates the technological progress in abatement that drives down the abatement cost. Similarly, we assume that the cost of CDR is decreasing over time with a rate that is proportional to the abatement cost coefficient. This proportion is five times the relative cost of removing 100% of emissions in year 2010 by using only abatement action. This way, we make sure that the cost of CDR always stays above the cost of abatement.

Part of the net output at each time step is saved and invested and the rest is consumed:

$$K_{t+1} = (1 - \delta) \times \ K_t + s \times \ Q_t \tag{A.6}$$

where δ is the capital depreciation rate and s is the saving rate. The industrial emissions E_t are found from the carbon intensity of output σ_t, taking into account the abatement decision:

$$E_t = \sigma_t \times (1 - a_t) \times \ Y_t \tag{A.7}$$

The atmospheric and ocean carbon concentrations in the next time period are:

$$M^{at}_{t+1} = M^{at}_t + E_t - r_t - N_t + O_t \tag{A.8}$$

$$M^{lo}_{t+1} = M^{lo}_t + N_t - O_t \tag{A.9}$$

where N_t is the ocean uptake and O_t is outgassing from the oceans. Similar to the theoretical model, the ocean uptake can be viewed as a fraction of current concentration in the oceans:

$$N_t = \beta_0 \times M^{lo}_t \tag{A.10}$$

The outgassing is a function of the level of CDR and its effectiveness (ν_t) which depends on the existing stock of concentration in the atmosphere:

$$\nu_t = \gamma_0 + \gamma_1 \times M^{at}_t \tag{A.11}$$

$$O_t = (1 - \nu_t) \times r_t \tag{A.12}$$

where γ_0 and γ_1 are the CDR effectiveness coefficients. The temperature equations for the next state are:

$$T^{at}_{t+1} = T^{at}_t + \eta_1 \times \left\{F_{t+1} - \eta_2 T^{at}_t - \eta_3 \times \ \{T^{at}_t - T^{lo}_t\}\right\} \tag{A.13}$$

$$T^{lo}_{t+1} = T^{lo}_t + \eta_4 \times \ \{T^{at}_t - T^{lo}_t\} \tag{A.14}$$

$$F_{t+1} = \eta_2 \times \log_2 (M^{at}_t / M^{at}_0) \tag{A.15}$$

where $\eta_1, ..., \eta_4$ are temperature coefficients.

- **Reward Function**
  The reward is calculated as the social utility of consumption at each time epoch:

$$U_t = L_t \times \left[\frac{\left\{(1 - s) \times \frac{Q_t}{L_t} \times 1000\right\}^{1-\alpha} - 1}{1 - \alpha} - 1 \right] \tag{A.16}$$

where α is the elasticity of marginal utility of consumption. The objective is to maximize the sum of discounted expected social utilities over the modeling horizon:

$$\max_{a_t, r_t} \sum_{t=0}^{T} \gamma^t \ U_t\big(S_t, a_t, r_t\big) \tag{A.17}$$

B Cost Effectiveness Analysis (CEA)

In the main text we conducted a cost-benefit analysis (CBA) for the derivation of the optimal solutions and the dynamics of our problem. In this section we will perform a cost-effectiveness analysis (CEA) as an alternative to cost-benefit analysis (CBA). Assume the case of climate stabilization policy, by the specification of a maximum level of carbon stock in the atmosphere $\bar{M}$, which can be translated into policy maker's goal for the level of pollution. In this case we can write our maximization problem by using the Lagrange formulation as

$$\mathcal{L} = E_1 - \frac{1}{2} a E_1^2 - \frac{1}{2} c_R R_1^2 - \frac{1}{2} \delta_M \bar{M}^2 + \lambda(\bar{M} - (M_1 + E_1 - \beta_0 S_1 - (\gamma_0 + \gamma_1 M_1) R_1)) \tag{B.1}$$

From the FONCs:

$$\frac{\partial \mathcal{L}}{\partial E} = 0 \Rightarrow E^*_t = \frac{1 + \lambda}{a} \tag{B.2}$$

$$\frac{\partial \mathcal{L}}{\partial R} = 0 \Rightarrow R_t^* = -\frac{(\gamma_0 + M_1\gamma_1)\lambda}{c_R} \tag{B.3}$$

and the constraint

$$\bar{M} = (M_1 + E_1 - \beta_0 S_1 - (\gamma_0 + \gamma_1 M_1)R_1) \tag{B.4}$$

Solving the system of the optimal emissions (E_t^*) and CDR level (R_t^*) along with the shadow cost of an increase in the budget for the carbon stock of the atmosphere (λ), we have

$$E_t^* = \frac{(\gamma_0 + M_1\gamma_1)^2 + c_R(\bar{M} - M_1 + \beta_0 S_1)}{c_R + a(\gamma_0 + M_1\gamma_1)^2} \tag{B.5}$$

$$R_t^* = \frac{(\gamma_0 + M_1\gamma_1)(1 - a(\bar{M} - M_1 + \beta_0 S_1))}{c_R + a(\gamma_0 + M_1\gamma_1)^2} \tag{B.6}$$

and the shadow cost

$$\lambda^* = \frac{c_R(a(\bar{M} - M_1 + \beta_0 S_1) - 1)}{c_R + a(\gamma_0 + M_1\gamma_1)^2} \tag{B.7}$$

We know that $-1 < \lambda < 0$ and if we define $\hat{M}$ as $(\bar{M} - M_1 + \beta_0 S_1)$, then we have

$$-\frac{(\gamma_0 + M_1\gamma_1)^2}{c_R} < \hat{M} < \frac{1}{a} \tag{B.8}$$

From B.8 we conclude that the rate of the emissions being removed from CDR ($\hat{M}$) should be greater than the relative effectiveness over the cost of CDR and lower than the reduction in utility due to an extra unit of emissions.

By differentiating optimal emissions and CDR levels with respect to the linear CDR effectiveness coefficient (γ_1) we have

$$\begin{aligned} \frac{\partial E_t^*}{\partial \gamma_1} &= \frac{2c_R M_1 \left(1 - a\left(\bar{M} - (M_1 - \beta_0 S_1)\right)\right)(\gamma_0 + M_1\gamma_1)}{(c_R + a(\gamma_0 + M_1\gamma_1)^2)^2} \\ &= \frac{2c_R M_1 \left(1 - a\hat{M}\right)(\gamma_0 + M_1\gamma_1)}{(c_R + a(\gamma_0 + M_1\gamma_1)^2)^2} \end{aligned} \tag{B.9}$$

$$\begin{aligned} \frac{\partial R_t^*}{\partial \gamma_1} &= -\frac{M_1 \left(1 - a\left(\bar{M} - (M_1 - \beta_0 S_1)\right)\right)\left(-c_R + a(\gamma_0 + M_1\gamma_1)^2\right)}{(c_R + a(\gamma_0 + M_1\gamma_1)^2)^2} \\ &= -\frac{M_1 \left(1 - a\hat{M}\right)\left(-c_R + a(\gamma_0 + M_1\gamma_1)^2\right)}{(c_R + a(\gamma_0 + M_1\gamma_1)^2)^2} \end{aligned} \tag{B.10}$$

From B.8 it is straightforward to show that $\frac{\partial E_t^*}{\partial \gamma_1} > 0$ and $\frac{\partial R_t^*}{\partial \gamma_1} > 0$. Thus, as the effectiveness of CDR increases (outgassing decreases) the optimal strategy is to increase emissions and CDR effort.

C Model with ocean acidification

High ocean carbon concentrations result in ocean acidification, which can lead to damages (Brander et al. (2012)). High atmospheric carbon concentrations may yield benefits (Pongratz et al. (2012)) or damages (Bony et al. (2013)). These damages are mostly unknown, and therefore this calibration must be rather arbitrary. In this section we extend our model to account for ocean acidification. We assume an additional type of damages related to the use of CDR methods and we can define the damages from ocean acidification as

$$D(S_t) = \frac{1}{2}\delta_S S_t^2 \tag{C.1}$$

where δ_S represents the marginal damages from ocean acidification. The carbon cycle consists in this case by two layers, one for the atmosphere and one for the ocean

$$M_{t+1} = M_t + E_t - R_t - (\gamma_0 + \gamma_1 R_t)\, S_t \tag{C.2}$$

$$S_{t+1} = S_t + (\gamma_0 + \gamma_1 R_t)\, S_t \tag{C.3}$$

The maximization problem is

$$\underset{E_1,R_1}{max}\, W\,(E_1, R_1) = E_1 - \frac{1}{2}aE_1^2 - \frac{1}{2}c_R R_1^2 - \frac{1}{2}\delta_M M_2^2 - \frac{1}{2}\delta_S S_t^2 \tag{C.4}$$

From the FONCs:

$$\frac{\partial W}{\partial E} = 0 \Rightarrow E_t^* = \frac{(1 - S_1\gamma_1)^2\delta_M + c_R(1 + y_1\delta_M) + S_1^2\gamma_1(\gamma_1 + (1 + \gamma_0 - (M_1 + S_1)\gamma_1)\delta_M)\delta_s}{\delta_M\,(c_R + S_1^2\gamma_1^2\delta_s) + a\,(c_R + (1 - S_1\gamma_1)^2\delta_M + S_1^2\gamma_1^2\delta_s)} \tag{C.5}$$

$$\frac{\partial W}{\partial R} = 0 \Rightarrow R_t^* = \frac{\delta_M(1 - ay_1)(1 - S_1\gamma_1) + S_1^2(1 + \gamma_0)\gamma_1(a + \delta_M)\delta_s}{\delta_M\,(c_R + S_1^2\gamma_1^2\delta_s) + a\,(c_R + (1 - S_1\gamma_1)^2\delta_M + S_1^2\gamma_1^2\delta_s)} \tag{C.6}$$

where $y_1 = \beta_0 S_1 - M_1$.

By differentiating optimal emissions and CDR levels with respect to (γ_0), we have:

$$\frac{\partial E_t^*}{\partial\gamma_0} = \frac{S_1\delta_M(c_R + S_1\gamma_1\delta_s)}{\delta_M\,(c_R + S_1^2\gamma_1^2\delta_s) + a\,(c_R + (1 - S_1\gamma_1)^2\delta_M + S_1^2\gamma_1^2\delta_s)} \tag{C.7}$$

$$\frac{\partial R_t^*}{\partial\gamma_0} = \frac{S_1(-a(1 - S_1\gamma_1)\delta_M + S_1\gamma_1(a + \delta_M)\delta_s)}{\delta_M\,(c_R + S_1^2\gamma_1^2\delta_s) + a\,(c_R + (1 - S_1\gamma_1)^2\delta_M + S_1^2\gamma_1^2\delta_s)} \tag{C.8}$$

By differentiating optimal emissions and CDR levels with respect to the correlation factor of outgassing and CDR (γ_1) we have

$$\begin{aligned}\frac{\partial E}{\partial\gamma_1} =& \frac{S_1\delta_M}{(\delta_M(c_R + S_1^2\gamma_1^2\delta_S) + a(c_R + (1 - S_1\gamma_1)^2\delta_M + S_1^2\gamma_1^2\delta_S))^2} \times \\ &\Big[((-2c_R L_5 L_1\delta_M - c_R S_1(1 + \beta_0)(-1 + 2S_1\gamma_1)L_3 \\ &+ S_1\delta_s(L_1(-2\gamma_1 + a(1 + \beta_0 - (2M_1 + S_1)\gamma_1 + S_1\beta_0\gamma_1))\delta_M - L_4\gamma_1^2)))\Big]\end{aligned} \tag{C.9}$$

$$\begin{aligned}\frac{\partial R}{\partial\gamma_1} =& \frac{1}{(\delta_M(c_R + S_1^2\gamma_1^2\delta_S) + a(c_R + (1 - S_1\gamma_1)^2\delta_M + S_1^2\gamma_1^2\delta_S))^2} \times \\ &\Big[(-2S_1(-aL_1\delta_M + S_1\gamma_1 L_3)(L_5 L_1\delta_M + L_4\gamma_1) + (-S_1 L5\delta_M + L_4)(\delta_M(c_R + S_1^2\gamma_1^2\delta_s) \\ &+ a(c_R + L_1^2\delta_M + S_1^2\gamma_1^2\delta_s))\Big]\end{aligned} \tag{C.10}$$

where $(1 - S_1\gamma_1) = L_1,\ a\,(M_1 - S_1\gamma_0) = L_2,\ (a + \delta_M\delta_S = L_3,\ {S_1}^2\,(1 + \gamma_0)\,L_3 = L_4\ and$ $1 + L_2 = L_5$. It is obvious from the previous complex formulas that clear signs cannot be derived for the dynamics in the theoretical model. Thus we continue our analysis by using the numerical DICE model.

C.1 DICE model with CDR and ocean acidification

In the basic DICE model, climate change damages are a quadratic function of global temperature only. Since SGE reduces temperatures but not atmospheric or ocean carbon concentrations, we can modify our model to account for damages from temperature and from ocean carbon concentrations. We keep the total level of climate change damages identical to the calibrated level in DICE. We assume that the majority (50%) of climate change damages come directly from temperature, and 30% of damages may come from ocean concentrations, and the remaining 20% are from atmospheric concentrations.

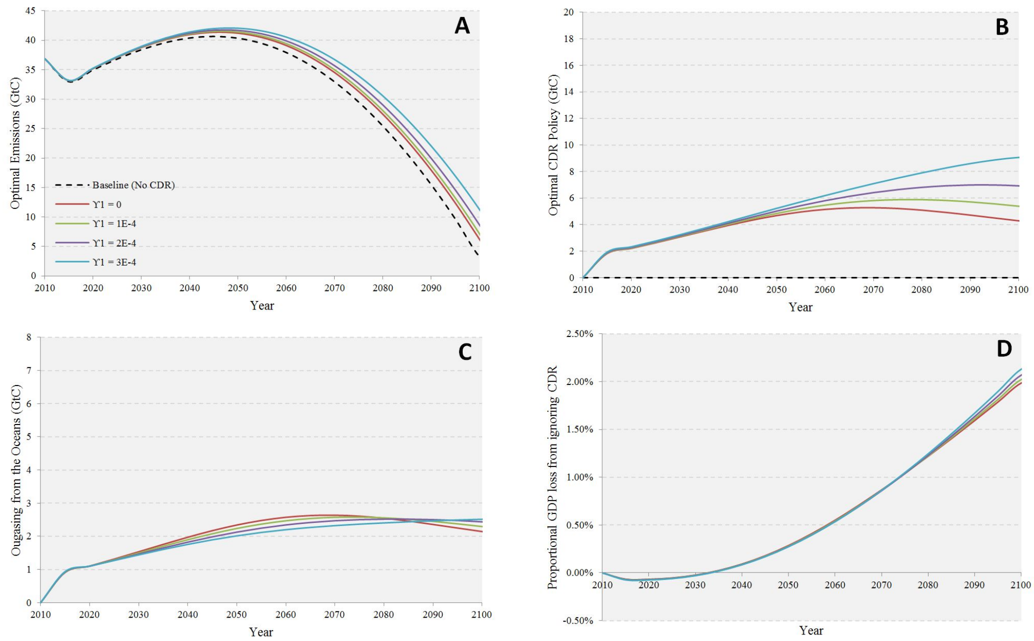

Figure C.1: Climate outcomes under different values of the outgassing parameter for the model with ocean acidification.

The results of this model have shown in Figure C.1. Inclusion of ocean acidification damages in the model reduces the deployment of CDR and to some extend abatement. It also changes the way outgassing impacts the optimal policy. Outgassing as discussed in the main part of this paper, reduces as the effectiveness of CDR increases. However and since outgassing reduces the ocean concentration of carbon, it alleviates the damages from ocean acidification. In panel **A**, optimal emissions increase as the effectiveness of CDR increases. Similar effect can be observed in the optimal CDR path shown in panel **B**. In **C** outgassing from the case with $\gamma_1 = 3E - 4$ reaches to about 2.2 GtC about half of the level it reached under the similar case but without ocean acidification. Since deployment of CDR and its outgassing effect helps reduce the damages both from both atmospheric and ocean concentrations, the cases with CDR has a clear welfare advantage to the case without CDR as shown in panel **D**.

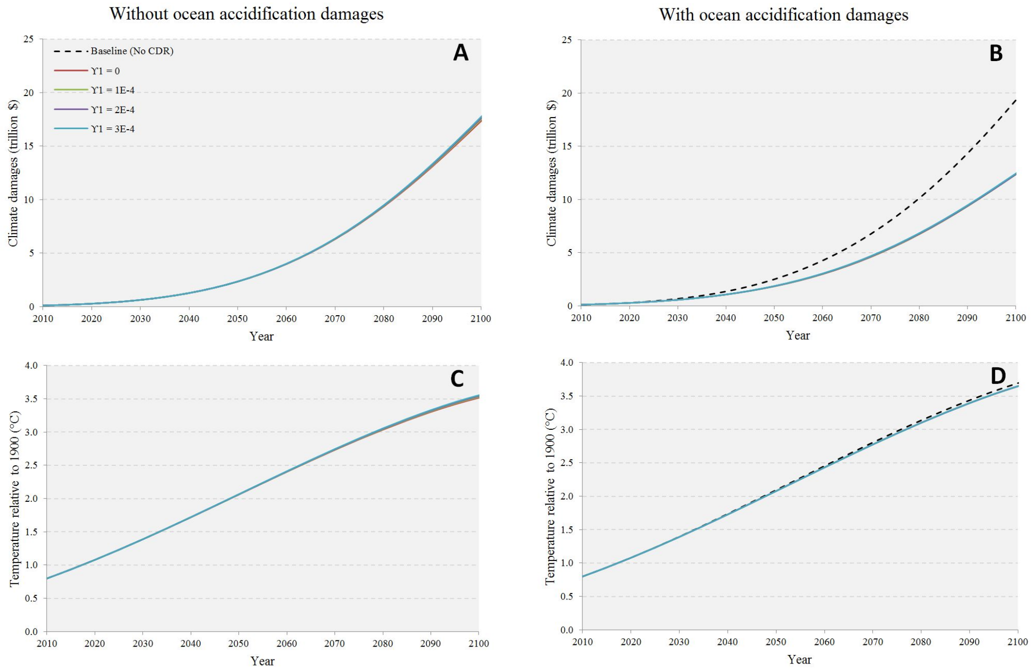

Figure C.2: Climate damages and atmospheric temperature under different values of the outgassing parameter for both models with and without ocean acidification. The left column shows the result for the main model without ocean acidification. The right column shows the result for the model with ocean acidification

Figure C.2 shows the total damages and atmospheric temperature in both models without and with ocean acidification. Panel **A** shows the damages in the main model without ocean acidification. Climate change damages are very close in all scenarios. In panel **B** with ocean acidification damages, CDR deployment reduces damages significantly. In this case, the damages are much lower than those in Panel **A** and outgassing has a major impact in this reduction. As outgassing increases, there will be less ocean acidification and therefore, less total climate damages. Panel **C** shows the temperature under different outgassing scenarios for the main model without ocean acidification. In all cases temperature increases gradually and peaks just under $3.5°C$. Higher coefficient of outgassing results in higher concentration of carbon in the atmosphere and therefore higher temperature. Panel **D** shows the temperature under different outgassing scenarios for the main model with ocean acidification. In this case the temperature is slightly higher since abatement level is reduced due to lower contribution of temperature in overall damages.

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